Motion of Vapor Bubbles in Saturated Liquids

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Mendelson (1) has proposed a new expression for the terminal velocity of rising bubbles in inviscid liquids by speculating that "the bubbles may be thought of as merely interfacial disturbances, whose dynamic behaviors should be similar to those of waves on an ideal fluid."

$$V_\infty = \left[\frac{2\sigma}{D_e \rho} + \frac{gD_e}{2} \right]^{1/2} \tag{1}$$

Peebles and Garber (2) have correlated rise velocities for bubbles in the range 0.14 cm. < D_e < 0.6 cm. by the expression

$$V_\infty = 1.35 \left[\frac{2\sigma}{D_e \rho} \right]^{1/2} \tag{2}$$

while Haberman and Morton (3) have shown that for bubbles with equivalent diameters greater than 0.6 cm., the rise velocity is given by

$$V_\infty = 1.02 \left[\frac{gD_e}{2} \right]^{1/2} \tag{3}$$

If Equations (1) through (3) are substituted into the expression for the drag coefficient

$$C_D = \frac{4g(\rho_l - \rho_v) D_e}{3 \rho_l V_\infty^2} \tag{4}$$

which represents the ratio of buoyancy to drag force acting on a rising bubble, the resulting expressions are given by

$$C_D = \frac{8}{3} N_{E_0} [4 + N_{E_0}]^{-1} \tag{5}$$

$$C_D = 0.366 N_{E_0} \tag{6}$$

$$C_D = \frac{8}{3} \tag{7}$$

respectively, where the Eötvös number is defined as

$$N_{E_0} = \frac{g(\rho_l - \rho_v) D_e^2}{\sigma} \tag{8}$$

and represents the ratio of buoyancy to surface forces. It is apparent that Equation (5) approaches Equation (7) as the Eötvös number becomes large and reduces to Equation (6) as the Eötvös number becomes small. The constant in Equation (6) was determined experimentally by Peebles and Garber using a relatively small diameter tube, and may be somewhat in error.

Data for vapor bubbles rising through saturated liquids have been obtained experimentally for a variety of liquids under pressures ranging from 48 to 540 mm. Hg. The equivalent diameters of these bubbles ranged from 0.1 to 1.3 cm. and thus cover precisely the region in which Mendelson's analysis should be applicable.

The data are compared with Equations (5) through (7) in Figure 1. Although considerable scatter of the data is evident, it is apparent that the mean of the data is in agreement with Mendelson's analysis. Also shown in Figure 1 is an additional expression proposed by Peebles and Garber for bubbles with equivalent diameters > 0.6 cm.

$$V_\infty = 1.18 \left[\frac{\sigma g}{\rho} \right]^{1/4} \tag{9}$$

or in terms of the drag coefficient

$$C_D = 0.958 N_{E_0}^{1/2} \tag{10}$$

Equation (9) has been used, for example, by Zuber (4) in determining a relationship between the frequency of bubble formation on a heated surface and the diameter of a vapor bubble at departure from the heated surface. It is apparent from Figure 1 that the data obtained here are not in agreement with Equation (9) or (10).

NOTATION

C_D = drag coefficient, defined by Equation (4), dimensionless

D_e = equivalent diameter of bubble, cm.

g = gravitational acceleration, cm./sec.^{2}

N_{E_0} = Eötvös number, defined by Equation (8), dimensionless

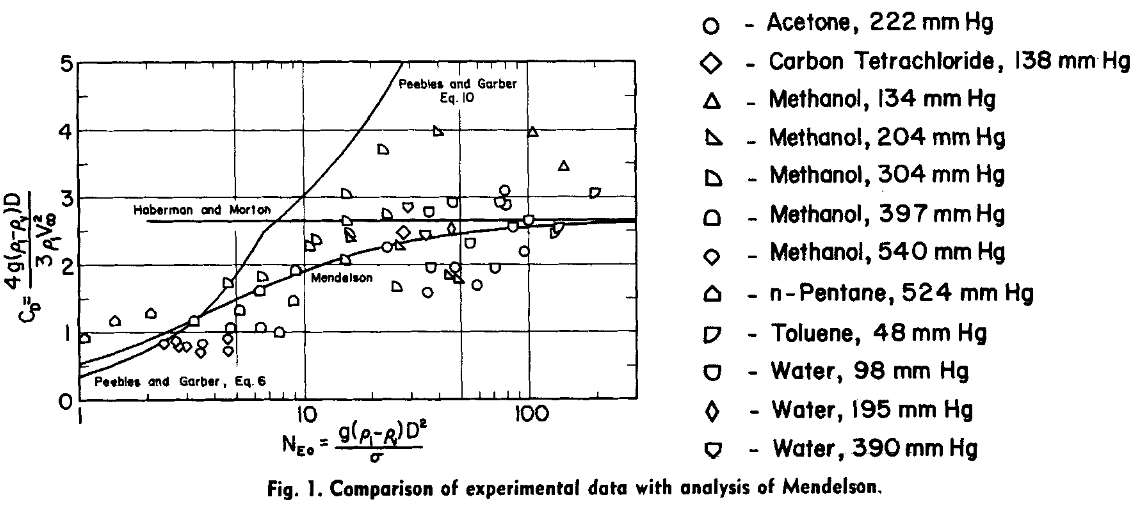

Fig. 1. Comparison of experimental data with analysis of Mendelson.

V_∞ = bubble terminal velocity, cm./sec.
ρ = density, g./cc.
σ = surface tension, dynes/cm.

Subscripts

l = liquid
v = vapor

LITERATURE CITED

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3. Haberman, W. L., and R. K. Morton, *David Taylor Model Basin NR 715-102* (Sept., 1953).
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A Mathematical Analysis of the Surface Temperature Variation in Heat Transfer Experiments

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McAdams (*1*) has presented a summary of theoretical and experimental information concerning the transfer of heat between a circular tube and a fluid flowing normal to the axis of the tube. The experimental data are correlated as average heat transfer coefficients defined in terms of the total heat flux per unit area and the average temperature difference between the surface and the bulk fluid. However, for tubes of large wall thickness or low conductivity, the surface temperature variation can be significant with respect to the maximum temperature difference between the surface and the bulk fluid, and thus the average temperature difference can be experimentally uncertain.

In general the calculation of the temperature variation along a tube surface, when the thermal resistance of the tube wall is comparable to the thermal resistance of the fluid, requires the solution of the heat conduction equation in both phases. As a first approximation to the solution, the thermal resistance of either phase may be assumed to be zero with the result that the surface temperature is found to be constant. As a second approximation, the thermal resistance of the fluid may be assumed to be nearly zero and may therefore be represented by a position-dependent surface heat transfer coefficient. The problem then is reduced to one of heat conduction in the tube wall for an arbitrary heat transfer coefficient. The solution to this problem is given for a tube with a heat generating core.

Carslaw and Jaeger (*3*) have given the solution to some steady heat conduction problems with position-dependent boundary conditions; however, an alternate series solution may be found if the position-dependent coefficient is expanded in the angular parts of the terms constituting the general solution of Laplace's equation.

THE FORMAL SOLUTION

The problem is to find the steady temperature distribution in an infinite hollow cylinder of radius R_2, containing a heat generating core of radius R_1, for any preassigned local heat transfer coefficient (Figure 1). The coefficient $h(\theta)$ is permitted to vary with circumference along the surface $R = R_2$ and in general is given by

$$h(\theta) = \sum_{p=0}^{q} a_p \cos p\theta \tag{1}$$

where q is any positive integer and where the flow direction is $\theta = 0$.

Laplace's equation for steady conduction may be written in cylindrical coordinates as

$$\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = 0 \tag{2}$$

when

$$\frac{\partial T}{\partial Z} = 0$$

The boundary conditions express the symmetry of the problem and the requirement that the flux be continuous at the surfaces $R = R_1$ and $R = R_2$. These conditions are given by

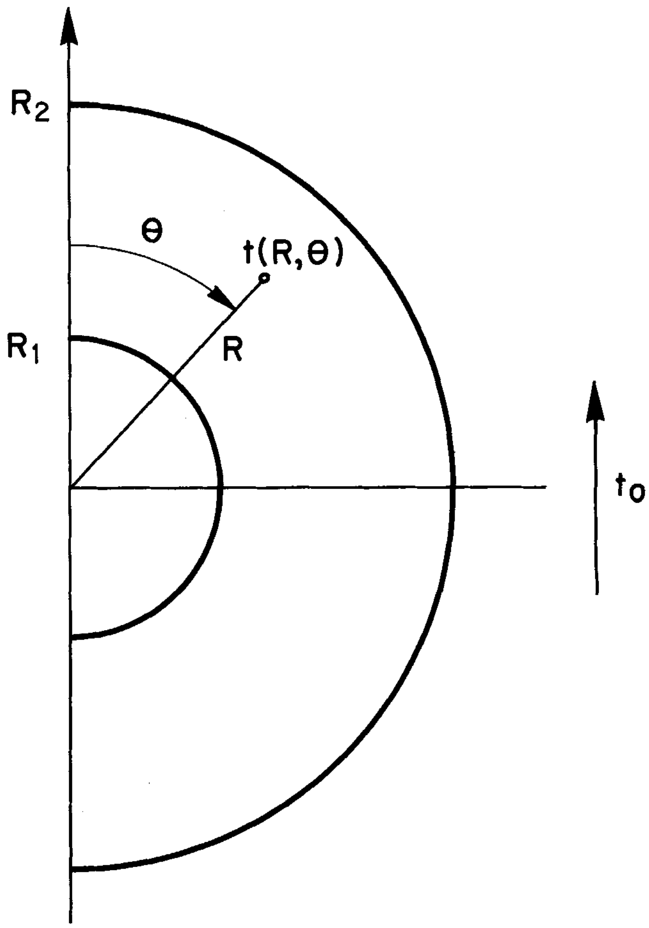

Fig. 1. Cross section of infinitely long hollow cylinder.